Inclined Plane

Equipment

- Calculator, Computer
- Air Track, Air Pump
- Air Track Cart
- Stopwatch

Objectives

- *To determine the acceleration of an object moving along an incline*
- *To use the acceleration of an object to determine the gravitational acceleration g.*

Introduction

In an earlier experiment, you attempted to use a stopwatch to measure the gravitational acceleration g by timing the fall of a dropped object. It was almost impossible to get a good result by this method. In the same lab, you used a computer to time the fall of another dropped object (the "picket fence") and found that by improving the timing method, a better measurement of g was obtained. In this lab, we'll go back to using the stopwatch, but we'll slow down the action by having our object float down a gentle incline instead of dropping straight down.

You already know the basics of kinematics. When an object undergoes one-dimensional uniformly accelerated motion, its velocity increases linearly with time. If the initial velocity of the object is zero at time $t = 0$, then the velocity v at any time t is given by the following expression:

$$v = at \tag{1}$$

Here, a is the acceleration, which is constant in magnitude and direction.

Consider a time interval between $t = 0$ and any later time t. The velocity is zero at $t = 0$, and the velocity at time t is v. With uniform acceleration, the average velocity v_{avg} during the time interval is:

$$v_{\text{avg}} = \frac{0 + v}{2} = v/2 \tag{2}$$

The displacement x of the object during the time interval t is given by:

$$x = v_{\text{avg}}t = \tfrac{1}{2}vt = \tfrac{1}{2}at^2 \tag{3}$$

Thus, Equation 3 states that if an object is released from rest, its displacement is directly proportional to the square of the elapsed time.

When an object is dropped, we assume **free fall**, which means its acceleration is g in a downward direction. A closely related situation is an object gliding down a slope. In this case, the acceleration is reduced by a factor determined by the angle of the slope. This results from the fact that the force of gravity $\vec{F}_g$ is a vector which can be resolved into two components, one component is perpendicular to the plane and another component is parallel to the plane. The perpendicular component is offset by the support force from the plane. The parallel component $(F_g \sin\theta)$ pulls the cart down the plane. The magnitude of the acceleration is then calculated from 2nd Law of Motion:

$$ma = mg\sin\theta \text{ or}$$
$$a = g\sin\theta \tag{4}$$

Here, θ is the angle of the slope, measured with respect to a horizontal surface. So if $\theta = 0$, the object doesn't accelerate. If $\theta = 90°$, the slope is vertical and the object falls freely. If θ is some small angle, then the acceleration will be small.

Experimental Setup

A cart is placed on an air track raised at one end to form an inclined plane whose angle of inclination is θ. Since measuring small angles is difficult, we will instead measure some distances and use them for the calculation of sin. Since $\sin\theta = \frac{\text{Opposite}}{Hypothenuse}$, the difference in height Δh between two random points along the slope can serve as the opposite side to the angle θ, and the distance between these points (along the slope) can serve as the hypotenuse.

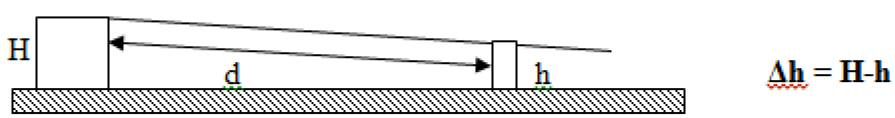

Figure 1. Slope showing the measured parameters needed to calculate the angle.

Then, $\sin\theta = \frac{\Delta h}{d}$.

When released, the cart glides down the inclined plane with acceleration $a = g\sin\theta$.

Data Recording

Using the stopwatch, you will measure the time it takes for the cart to slide various distances. Later, during the analysis, you will determine the acceleration a and then the gravitational acceleration g.

Please do not draw marks on the air track. If you want reference marks, use a Post-it. Make sure the Post-it is low enough that the cart doesn't touch it as it glides by.

1. Hook up the air track to the pump, place the cart on it, and test it out. Make sure the cart can glide freely back and forth.
2. Choose two points along the track and measure their heights above the table and the distance d between them. *These values are vital to the success of the experiment, so you probably want each partner to verify the measurements.*
3. Have one partner hold the cart at rest near the top of the incline. Find the point 0.25 m (25 cm) down the track. Have another partner get ready with the stopwatch.
4. Release the cart from rest and time how long it takes for the cart to glide the first 0.25 m down the track.
5. Repeat the measurement for five (5) trials at this distance.
6. Repeat the experiment for distances of 0.50 m, 0.75 m, 1.00 m, and 1.40 m.

Δx (m)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)	t_{avg} (s)	$(t_{\text{avg}})^2$ (s²)
0.25							
…							

Table 1. Measurements of the time for an air cart to slide various distances down a track, when released from rest.

Data Analysis

When we expect a proportional relationship like that between Δx and t^2, one way of finding the proportionality constant (in this case, $\frac{1}{2}a$) is to calculate its value for each data point (i.e. each row in the table). The problem is that some kinds of experimental errors will keep us from ever getting a good value of a. Even if we take the average of all of our a values (one from each row), the error can still linger. One way of eliminating some kinds of errors is to use the **linear best fit** to measure of the proportionality constant. The slope of the trend line works great for this purpose.

1. Make a scatter plot of Δx vs. t^2. (Hint: vertical axis vs. horizontal axis)
2. Add the linear trend line and its equation. Record the slope.
3. Calculate the acceleration a from the slope. (Hint: $\Delta x = \frac{1}{2}at^2 = (\text{Slope})t^2$.)
4. Find the acceleration due to gravity from your cart's acceleration.
5. In the abstract, comment on the accuracy of the experiment and discuss possible causes of the error.

Description	Value
Slope (m/s^2)	
Acceleration a (m/s^2)	
Height Difference Δh (m)	
Distance between points d (m)	
$\sin\theta$	
$g_{\text{experimental}}$ (m/s^2)	
% Error	

Table 2. Various values calculated from the measurements, ultimately producing an experimental value of the gravitational acceleration g.

Requirements for the Report (also consult the rubric):

Save your Excel files through your Blackboard Group File Exchange

- The **abstract section** must contain the following explanations in paragraph form:
  - How the data was collected and calculated for Table 1
  - How the data from Table 1 was analyzed including interpretation of the trendline in Figure 1 (Δx vs. t_{avg}^2, why time was squared?)
  - How the data was collected and calculated for Table 2 (including formulas)
  - How the data from Table 2 was analyzed by calculation of % error
  - A general statement based off Table 2 about why the experimental value of g differs from the accepted value of 9.81 m/s^2 (mention at least one specific error in the execution of the experiment or a factor that was negated in the calculations that could have led to this difference. This error or factor should follow logically from your experimental g value. For example: if your experimental value is greater than 9.81m/s^2, do NOT cite friction as being responsible for the % error as this does not follow logically from your g value).
- The **data section** must include
  - 2 Tables (labeled and captioned)
  - 1 Graph (titled, axis labels, axis units, labeled and captioned)
    - Δx vs t_{avg}^2